

Stability analysis of the banking system: a complex systems approach

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Abstract. The present work deals with the stability analysis of a banking system with the structure in the form of Apollonian graph based on such characteristics of the banking system as the modularity and inhomogeneous distribution of banks by degree, on the basis of the extended mean-field Nier model (a static approach based on a simplified balance sheet of assets and liabilities of the bank) which was used to analyze the extent of the process of bankruptcy of banks after the default of one of the banks in the banking system. The obtained results of research of stability of banking systems based on the Apollonian graphs indicate that such characteristics as modularity (i.e. clustering), and the heterogeneity of banks in the structure of the model of banking systems allow them to conform «isomorphous structure» typical of the majority of real social and biological complex adaptive systems.

Keywords: banking and finance, financial stability, complex systems, financial contagion, network approach.

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1. INTRODUCTION

In the recent few years, important steps have been taken towards a deeper understanding of critical phenomena in complex networks (Barabasi, 2015; Newman, 2010; Cohen & Havlin, 2000; Strielkowski, 2017), in particular, dynamic processes in the networks (Barrat et al., 2008), which can be of significant importance in the study of the mechanism of functioning and control of real processes such as social systems, the spread of viruses and epidemic diseases in computer networks, traffic in technological information systems, as well as in energy systems and networks (Brodzicki, 2015; Cieřlik et al., 2016; Balitskiy et al., 2016). For the latter process, such indicators as the probability of infecting the vertices of a complex network, the existence of nonequilibrium phase transitions, and the identification of the type of transition are important for studying the mechanism and, accordingly, the prognosis of the spread of epidemic processes (Pastor-Satorras et al., 2015) or migration flows (Čajka et al., 2014; Strielkowski et al., 2016).

Complex networks describe many natural and public systems (Pastor-Satorras & Vespignani, 2004), and, in most of them, three characteristic features are observed: the power-law distribution of the network vertices by degree, short average path length and high clustering coefficient. As for the topology of complex networks, it is usually divided into three large classes: random networks (Erdős-Rényi networks, ER) in which all nodes are connected to each other in randomly; scale-free networks (Barabási-Albert networks, BA) representing a connected graph with a power-law distribution of nodes by degree of connectivity; and networks with the effect of "small world" (Watts-Strogatz networks, WS), in which the diameter of the network or the mean shortest path length l grows logarithmically with the growth of the system size N (the number of nodes).

In studies on the behavior of complex networks, the main focus is on the analysis of the relationship between the reliability of a complex network and its topology, since its elements, that is, the network nodes and connections between them largely influence the behavior of the network (Matisziw et al., 2012). In the paper (Albert et al., 2000) it is shown that the topological characteristics of the network have a significant effect on its reliability. Further studies have shown that the scale-free complex of Barabási-Albert networks, which take into account the heterogeneity of the nodes and their power-law distribution by degrees are more resistant to random attacks than the homogeneous Erdős-Rényi networks, however, at the same time they are more fragile to targeted, intentional attacks (Cohen et al., 2001; Sun et al., 2007). On the other hand, homogeneous Erdős-Rényi networks, with a uniform distribution of nodes by degrees, are sufficiently resistant to targeted attacks (Cohen & Havlin, 2010; Yuan et al., 2015).

It should also be noted that many real natural and artificial systems exhibit a modular structure where nodes in small groups (so-called modules, clusters or communities) are more tightly connected to each other than the nodes in different modules of the network, which is the key to their behavior and functioning of the whole system (Girvan & Newman, 2002). The interrelationships between modules are relatively rare (as compared to a dense network of links within modules) but they are crucial to the functionality of the entire system, and quite often, the failure or breakdown of these links leads to a reduction or even a total malfunction of the system. Modular organization of the Internet and other large-scale infrastructures drastically increases their scalability and significantly influences the diffusion processes in them (Eriksen et al., 2003). Modules composed of protein complexes and dynamic functional units form the building blocks of molecular networks (Guimera et al., 2004). Based on the modular hierarchical principle, important problems of theoretical economics are being solved (the problem of choice; the transition from the micro to macro level of the economy; the problem of accounting for psychological factors and others) that are not solved within the mainstream direction (Spirin & Mirny, 2003). Finally, it is believed that it is the modular architecture of neural networks of the brain that is crucial for the functional requirements for the segregation and integration of information (Maevskii & Chernavskii, 2003; Bassett & Gazzaniga, 2011; Bullmore &

Sporns, 2009). In addition, an important fact was established: a modular structure appeared in natural systems due to multicriteria optimization of stability, efficiency and ability of the system to grow (Gallos et. al., 2012).

Thus, by changing the distribution of nodes by degrees it is possible to change the stability of complex networks. It was established in the work (Pan & Sinha, 2008) that scale-free networks with an "onion structure" are very robust to targeted attacks on high-degree nodes. Later, this result was confirmed by other researchers, and this discovery was developed to elaborate and design reliable and stable complex networks (Herrmann et. al., 2011; Wu & Holme, 2011; Tanizawa et. al., 2012). In recent works, the causes and mechanism of this effect have been identified (Zeng & Liu, 2012; Sun et. al., 2015).

2. FINANACIAL NETWORKS

Obviously, the listed properties of natural complex networks are also characteristic of complex financial systems (FS), including the banking system (BS). The problem is to identify the influence of structural characteristics of the network on its financial stability. In order to obtain a deeper understanding of the impact of the financial system architecture on its reliability, most studies use numerical and visual methods of network analysis, for example, in the works (Li et. al., 2015; Leon & Perez, 2014; Leon et. al., 2013), which analyzed the national payment and settlement systems of Colombia were undertaken. They confirmed that these local financial networks tend to self-organize in the form of a modular (i.e. clustered) scale-free (i.e. heterogeneous) architecture.

The obtained numerous research results show that local financial networks self-organize into modular, scale-free structures similar to an isomorphic architecture that is characteristic of other social and biological systems and is well known. The existence of an isomorphic modular scale-invariant architecture of financial systems is not as much important as is its obvious contradiction with the traditional assumptions on the mechanism of formation and structure of the financial system. We can mention the four most important consequences that are directly related to financial stability (Leon & Machado, 2013). First, the presence of a modular scale-invariant FS architecture leads to the fact that the results of traditional modeling of financial systems based on their homogenization in the form of homogeneous systems (as was done in the famous paper (Leon & Perez, 2014)) can be misleading due to misunderstanding of the influence of the factor of connectedness in finding out the mechanism and dynamics in distribution of financial contagions. Secondly, such architecture is a new feature of a complex interaction and adaptive behavior of financial institutions, which contributes to the fact that the system adjusts itself in such a way as to promote the growth of routine reliability and performance, with a few exceptions leading to the system fragility, as well as to the evolution of the systems in full agreement with the famous definition of financial networks as "reliable, but fragile" (Allen & Gale, 2000). Third, the modularity of the network structure leads to the limitation of cascades and the isolation of feedbacks (Haldane, 2009; Haldane & May, 2011; Anderson, 1999), causing in cases of negligent reduction of the system heterogeneity, for example, as a result of simple reduction or dismantling of systemically important financial institutions (even for the sake of improving financial stability), it is possible to get the opposite result in the form of unpleasant consequences leading to the decrease in the reliability of the financial system and its greater exposure to crises. Fourth, the supervisory and control bodies need to understand and take into account these features of the existing FS architecture to develop and implement effective measures of macroprudential regulation, and to calibrate the system requirements, as suggested in the works (Leon et al., 2013; Haldane, 2009; Anderson, 1999; Kambhu, 2007; Markose, 2012).

The theoretical approach to explaining the modular scale-free architecture of financial networks was first considered in the studies on the behavior of complex adaptive systems by Anderson (Anderson P., 1999) and Holland (Holland, 1998) and was based on such network characteristics as the growth mechanism due to the preferential addition of new nodes to the existing network proposed in the work (Barabasi & Albert, 1999), in the presence of two competing feedback mechanisms in adaptive networks: homophilia and homeostasis. This theoretical approach to the FS structure combines, on the one hand, the adaptive nature of the behavior of financial institutions based on the individual features of the evolutionary process through the use of the selective method of "trials and errors", and, on the other hand, the process of self-organization that leads to a stable modular, scale-free architecture of the system that ensures, as a rule, the stability of routine behavior of the system, except for the cases of rare but strong shock effects.

3. APOLLONIAN NETWORKS

Among complex networks, Apollonian networks are special since they are characterized by the following features (Da Silva et. al., 2013):

1. they are scale-free – the cumulative distribution function of nodes by degree $P(k) = \sum_{k' > k} m(k', n) / N_n$ shows a power-law character, that is $P(k) \propto k^{1-\gamma}$ with the exponent $\gamma = 1 + \ln 3 / \ln 2$, and is characterized by stationarity of distribution. Here k is the degree of the vertex (connectedness), and $N_n = 3 + (3^{n+1} - 1) / 2$ is the number of vertices in each n generation;
2. they exhibit the "small world" effect with an average length of the shortest path between two vertices $l, l \propto [\ln(N)]^{3/4}$, which grows more slowly than any positive degree of the system of N size. The clustering coefficient in the limit of large N is $C = 0.828$ (since l grows logarithmically and C tends to unity, as in the case of a regular lattice, the Apollonian network indeed demonstrates the "small world" effect).

The theoretical approach based on Apollonian graphs was used to describe the behavior of many physical or biological systems, namely: bond percolation, magnetic systems, neural network activity studies, vehicular traffic formation of public opinion.

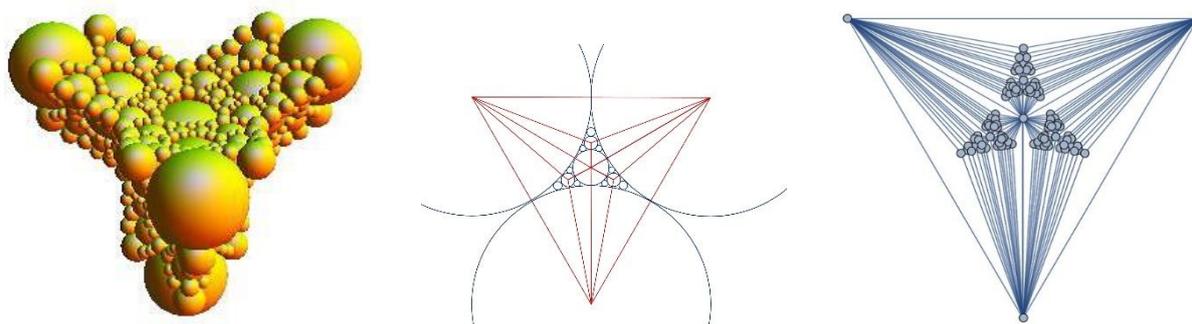


Figure 1. Three-dimensional (a) and two-dimensional (b) Apollonian graphs and a determinate Apollonian graph (c) generated by the GraphData ["Apollonian", 5] function of the Wolfram Mathematica system 10.3

The present work attempts to expand the use of Apollonian networks to study the spread of financial contagion in the interbank credit network using Nier's mean-field model. From the theoretical point of view, despite its simplicity, the mean-field model (MF) is the traditional method used to study the behavior

of such networks, since it describes qualitatively well the majority of phase transitions, in particular, the critical behavior of complex networks that belong to the MF universality class.

Deterministic Apollonian networks

Figure 1 shows the three-dimensional and two-dimensional deterministic Apollonian graphs (abbreviation –*DAN*), the construction procedure of which is described in detail in the works (Zhang et. al., 2006b).

In this paper, the GraphData function ["Apollonian", 5] from the library of the Wolfram Mathematica 10.3 system generating a graph of 105 nodes was used to construct deterministic Apollonian graphs (*DAN*), see Figure 1 (c).

Random Apollonian networks

For the construction of random Apollonian graphs (abbreviation – *RAN*), the iterative algorithm developed in the work (Zhongzhi Zhang, Lili Rong, 2006) was used: *RAN* [100, 3] generating a graph with 100 nodes.

Evolutionary Apollonian networks

To generate the evolutionary Apollonian graphs, the algorithm developed in the work (Zhang, et. al., 2006a) was used: *EAN* [4, 4, *p*] with different values of the parameter $p = \{0.1; 0.3; 0.5; 0.7\}$. Note that at $p \sim 0$, the *EAN* [4, 4, *p*] algorithm reduces to *RAN*, and at $p \sim 1$ – to *DAN*.

4. THE MODEL OF THE BANKING SYSTEM STABILITY WITH THE APOLLONIAN GRAPH STRUCTURE

The work studies the mechanism of bankruptcy spread of banks after the default of a randomly selected bank through the banking network with different topologies: deterministic (*DAN*), random (*RAN*) and evolutionary (*EAN*) Apollonian graphs (AG) taking into account different probability values to connect any nodes of the network $p = \{0.05; 0.1; 0.3; 0.5; 0.7; 0.9\}$, and taking into account the capital of the banks and the share of the interbank assets.

Results of computer experiments and their analysis

Table 1 presents numerical values of the model parameters and the range of their changes (the simulation was performed in the Mathematica 10 system, the codes of the software implementation of the model can be provided upon request).

Table 1

Numerical values of the model parameters and the range of their changes

Parameter	Network type	Definition	The range of changes
N_0	Deterministic AG(<i>DAN</i>) Random AG(<i>RAN</i>) Evolutionary AG(<i>EAN</i>)	Total number of banks	105 100 22; 69; 134; 172
E	For all	Total number of external assets	100000
y	For all	Share of the bank's capital in the total assets of the bank	$0 < y \leq 0.1$
Θ	For all	Share of interbank assets in total assets	$0 < \Theta \leq 0.5$
p	Evolutionary AG(<i>EAN</i>)	The probability of linking any two network nodes	0.05; 0.1; 0.3; 0.5; 0.7; 0.9

Influence of the network topology, capitalization of banks, share of interbank assets and connectedness of banks on the banking system instability

In this paper, as an assessment of systemic risk, the following criterion was used – bankruptcy of 10% of banks due to default of a randomly selected bank. This choice of assessing the systemic risk of the banking system is in good agreement with a similar assessment of systemic risk and analysis of the stability of the banking system (the transition of the banking system from regime I to regime II) following the default of a randomly selected bank that were carried out in May's work (May R., 2010). In the work by Gai and Kapadia (2010), systemic risk is defined as the bankruptcy of more than 5% of banks due to the bankruptcy of a randomly selected bank.

Like in the referred works (Karaev & Melnichuk, 2015; Karaev & Melnichuk, 2016; Kirsienė & Miseviciute, 2017), in which the analysis of the stability of the banking system with the topology in the form of Barabási-Albert (BA) and Watts-Strogatz (WS) graphs was undertaken on the basis of Nier's mean-field model, in this paper the results of computer experiments on the analysis of the instability of the banking system with the topology of the Apollonian graphs (*DAN*, *RAN*, *EAN*) are presented as:

1. the three-dimensional dependence of the number of bankrupt banks N on the numerical values of the parameters: the ratio of equity capital to the bank's assets γ and the ratio of the interbank assets to total assets Θ ;
2. the projection of the graph of three-dimensional dependence of the number of bankrupt banks N on the (γ, Θ) plane, for different levels $N/N_0 = \{0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}$ and $p = \{0.1; 0.3; 0.5; 0.7\}$ in the case of the banking system as an *EAN* graph.

The stability model of the banking system based on the Apollonian graphs (DAN, RAN, EAN)

In this paper, the analysis of the stability of the banking system with the topology of the Apollonian graphs (*DAN*, *RAN*, *EAN*) subjected to idiosyncratic shocks was carried out in the same way as in the famous work by Nier (Nier E. et. al., 2007), provided that the variation of the values of the model parameters does not cross the boundary of the range of values of the model parameters $\{0 < \gamma < 10\%; 0 < \Theta < 50\%\}$.

Deterministic (DAN) and random (RAN) Apollonian graphs

Figures 2 and 3 show the results of the banking system stability based on the deterministic (*DAN*) and random (*RAN*) Apollonian graphs subjected to idiosyncratic shock effects in the form of a default of a randomly selected bank, as: a) the three-dimensional dependence of the number of bankrupt banks N on the banks capitalization $\gamma\%$ and the share of interbank assets $\Theta\%$; b) the projection of the three-dimensional dependence of the number of bankrupt banks on the plane $N = 10\%N_0$ (the red line is the curve separating the stable and unstable phases of the banking system).

From the analysis of the obtained results presented in Figures 2 and 3 it follows that the curve separating the stable and unstable phases of the banking system subjected to shock in the form of a default of a random bank consists of two branches in the form of almost straight lines that intersect at a threshold point with the coordinates (γ_c, Θ_c) . The upward branch of the curve, which separates the stable and the unstable phases of the banking system, is characterized by the fact that with the growth of the share of interbank assets (the growth of the parameter Θ to the threshold value Θ_c) with a constant number of interbank bonds k to limit the scale of possible spread of bank failures and to maintain the stability of the banking system, the necessary level of capitalization needs to be increased ($\Theta \sim k\gamma$). The downward branch of the curve that divides the stable and unstable phases of the banking system is characterized by the fact that with a further increase in the share of the interbank assets (the growth of the parameter Θ above the threshold value Θ_c), the share of external assets falls, and consequently, the shock amplitude s impacts the banking system, which is the share, f , of external assets $s = f(1 - \Theta)$. Thereby, the scale of the possible spread of bank failures is reduced, and in order to maintain the stability of the banking system, the necessary

level of capitalization of banks also falls $y = f(1 - \Theta)/(1 + \kappa)$. The threshold level of the share of interbank assets Θ_c is determined from the condition of crossing of the ascending and the descending branches of the curve that separates the stable and unstable phases of the banking system $\Theta = (f - y)/(1 + f)$ (May & Arinaminpathy, 2010).

As can be seen from Figures 2 and 3, with the change in the topology of the banking system from *DAN* to *RAN*, there occurs the shift of the coordinates of the threshold point (y_c, Θ_c) for the banking system with the structure in the form of a deterministic Apollonian graph (*DAN*) – $\{y_c \sim 2\%; \Theta_c \sim 10\%\}$; while for the banking system with the structure in the form of a random Apollonian graph (*RAN*) it is $\{y_c \sim 4\%; \Theta_c \sim 30\%\}$. This displacement of the coordinates of the threshold point (y_c, Θ_c) can be explained by different values of the parameters f and κ for the banking network structure in the form of Apollonian graphs *DAN* and *RAN* at the same values of other model parameters: N_0, y, Θ . If we analyze the effect of the f/κ parameters ratio on the stability of the banking system, then, from Figures 2 and 3 it follows that for the banking system in the form of the *DAN* graph this ratio f_1/κ_1 is less than for the banking system in the form of the *RAN* graph – f_2/κ_2 . Assuming that the changes in the number of κ links more significantly affect the changes in the f/κ ratio than the changes in the f parameter; then it follows that for the banking network in the form of three *DAN* graph the number of κ_1 links is greater than the number of κ_2 links for the banking network in the form of the *RAN* graph. Thus, with the increasing number of connections in the κ graph, the weight of one bond – w – decreases. This leads to the fact that for the stability of the banking system on the upward branch of the curve that separates the stable and the unstable phases of the banking system at fixed values of the Θ parameter with increasing κ parameter the necessary level of the banks capitalization y to maintain the stability of the banking system reduces.

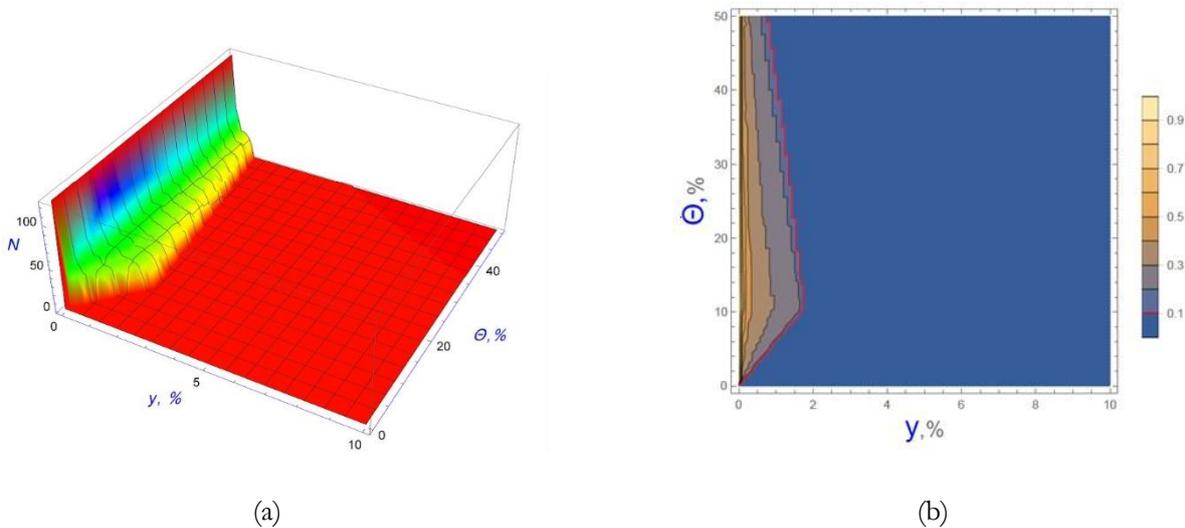


Figure 2. Deterministic Apollonian graph *DAN* (GraphData ["Apollonian", 5]):

a) the three-dimensional dependence of the number of bankrupt banks N on the capitalization of banks $y\%$ and the share of the interbank assets $\Theta\%$; b) the projection of the three-dimensional dependence of the number of bankrupt banks on the $N = 10\%$ N_0 plane (the red line is the curve separating the stable and unstable phases of the banking system).

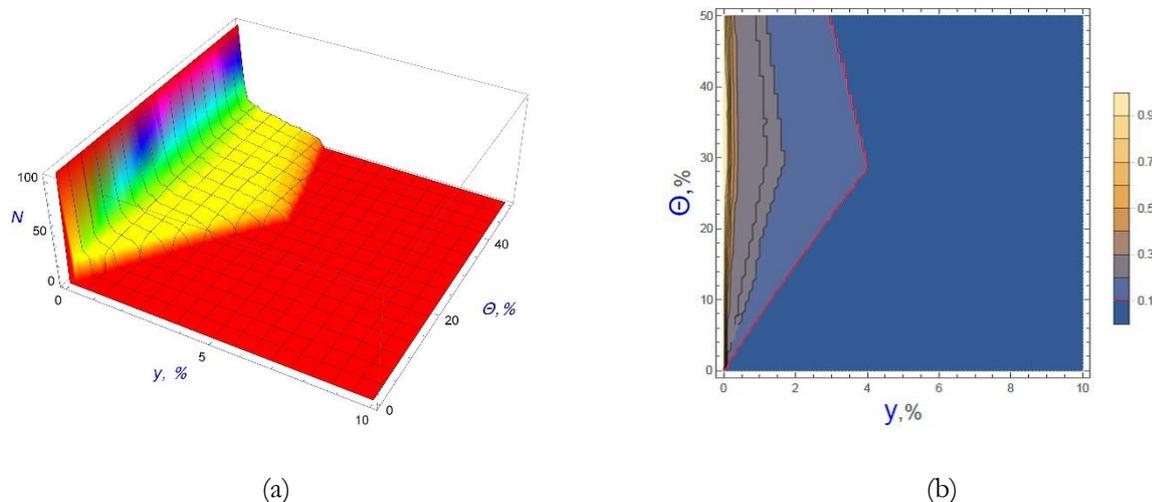


Figure 3. Random Apollonian graph RAN [100, 3]:

a) the three-dimensional dependence of the number of bankrupt banks N on the capitalization of banks y % and the share of the interbank assets Θ %; b) the projection of the three-dimensional dependence of the number of bankrupt banks on the $N = 10\%N_0$ plane (the red line is the curve separating the stable and the unstable phases of the banking system).

As for the stability of the banking system on the downward curve that separates the stable and unstable phases of the banking system, at the fixed values of the Θ parameter with the increasing values of the κ parameter, the share of the banking capital, y , to maintain the stability of the banking system is progressively lower since on this branch of the curve separating the stable and unstable phases of the banking system the growth of the share of the interbank assets leads to the decrease of the share of external assets, as well as to the growth of the capitalization of the banks. Therefore, in order to maintain the stability of the banking system with the increase in the κ parameter the required level of the banking capitalization becomes less and less.

The model of stability of the banking system based on the evolutionary Apollonian graph EAN

Figure 4 shows the phase diagram of the banking system based of the evolutionary Apollonian graph EAN with the probability of appearance of a connection between any nodes of the graph $p = \{0.1; 0.3; 0.5; 0.7\}$. It should be noted that, as can be seen from Figure 4, for extreme cases, when the probability of bond formation p tends to zero ($p \sim 0$), the EAN graph reduces to a random Apollonian graph RAN . In the case when the probability of bond formation p tends to unity ($p \sim 1$), the EAN graph reduces to the deterministic Apollonian graph DAN and occupies an intermediate position between them.

As can be seen from Figure 4, for the banking system based on the EAN graph, in the analyzed section $\{0 < y < 10\%; 0 < \Theta < 50\%\}$ for all values of the parameter $p = \{0.1; 0.3; 0.5; 0.7\}$, the boundary dividing the phase (stable and unstable) states of the banking system is represented by two straight lines: $\Theta = 1 - y(1 + \kappa)/f$ и $\Theta = \kappa y$. With the increase of p , the probability of formation of a connection between any nodes of the graph and at the same values of the Θ and y parameters, the parameter f/κ decreases, which is associated with the decrease in the amplitude s of shock impacts on the banking system (due to the growth of the κ parameter).

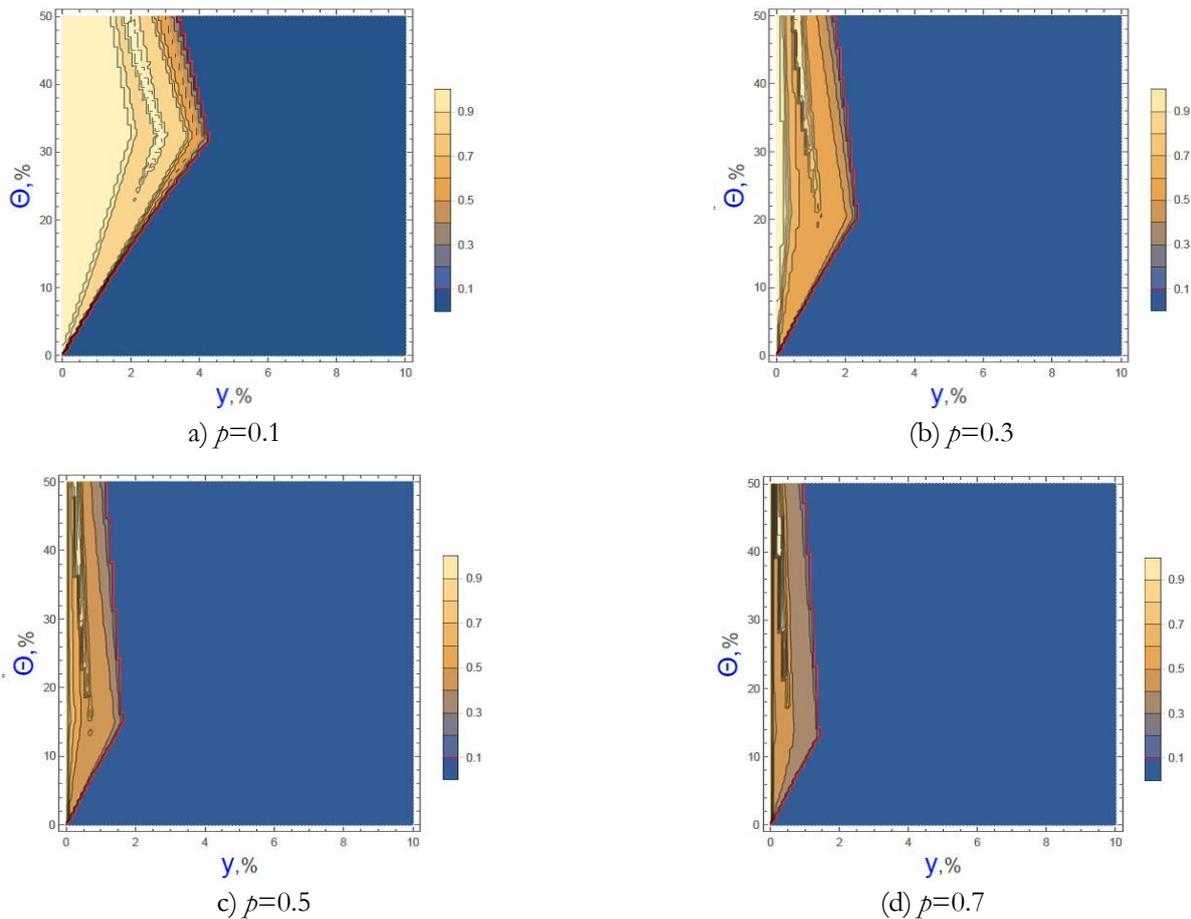


Figure 4. The stability of the banking system based of the evolutionary Apollonian graph $EAN[4, p]$, the projection of the three-dimensional dependence of the number of bankrupt banks on the $N = 10\% N_0$ plane (red line is the curve separating the stable and the unstable phases of the banking system) at different values of $p = \{0.1; 0.3; 0.5; 0.7\}$.

Thus, for banking systems with a structure in the form of an evolutionary Apollonian graph, the resistance to accidental shock effects in the form of a default of a single bank grows with the growth of the banks' connectedness with the banking system.

5. CONCLUSIONS

The paper analyzes the bankruptcy process after the default of one bank with the structure in the form of Apollonian graphs based on Nier's extended mean-field model (static approach based on the simplified balance of assets and liabilities of the bank).

In contrast to the cases of spread of the banks bankruptcy (able to cause systemic risk after the default of an individual bank) in the banking system:

- with a homogeneous distribution of banks by degree (Erdős–Rényi graph), which was considered in the majority of studies, including Nier's work proper (Nier et. al., 2007);
- on the bases of the Watts-Strogatz graph with the "small world" effect, with a high clustering coefficient and a short mean path length;

- with a non-uniform distribution of banks by degree – the model based on the Barabási-Albert graph (a scale-free network with a power-law distribution of banks by degree).

In this paper, the analysis of the stability of the banking system with a network structure in the form of an Apollonian graph has been undertaken, which is more closely approximated to the structure of a real banking system, taking into account such network features as modularity and heterogeneous distribution of banks by degree.

The obtained in this research results on the stability of the banking systems based on the Apollonian graphs suggest that the presence in the structure of model banking systems of such features as modularity (i.e. clustering) and heterogeneity of network nodes is analogous to the "isomorphic architecture" characteristic of most social and biological complex adaptive systems. It is not so much important the very existence of modularity and scale-invariant architecture of the banking systems, but the presence of the obvious contradiction with the results of traditional modeling of financial systems based on their homogenization in the form of homogeneous systems, as has been done in the famous research (Leon & Perez, 2014). This can be misleading due to the lack of understanding of the influence of the connectedness factor in the mechanism and dynamics in spread of financial contagion. In the cases of neglect or decline in the role of such system features as modularity and heterogeneity of system nodes, for example, as a result of simple reduction or dismantling of systemically important financial institutions (even for the sake of improving financial stability), the opposite result may be obtained in the form of unpleasant consequences leading to the reliability of the financial system and its greater exposure to crises. Therefore, the supervisory and control entities of the banking systems must take into account these features of the existing modular-hierarchical architecture of the BS in order to develop and implement effective measures of macroprudential regulation aimed at achieving financial stability. In this regard, the most effective strategy can be based on the results of studying biological systems, in particular, to develop effective policies aimed at improving the reliability and stability of the banking system on the basis of the theory of spread of epidemics. Recently there has been a growing awareness of the significant fact that not only the size of the financial institutions is important, but also the specificity of their interconnectedness with each other (Kravchuk, 2017; Bobrikova, 2017). The supervisory bodies need to use the network analysis of financial systems taking into account the modularity and heterogeneity of the system, not only to identify systemic and vulnerable institutions, but also to track potential contagious paths in the network.

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